

The thermodynamics of the drinking bird toy

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The popular drinking bird toy is analysed thermodynamically as a heat engine. The mechanical efficiency of this engine is shown to be much less than the thermodynamic efficiency. Simple and yet important experimental techniques are used in obtaining data, and powerful least-squares computer curve fitting techniques are used to extract parameters.

Many children are fascinated by the bird-shaped object which constantly dips into a glass of water to take a drink and is available in many toy and gift shops. In fact this toy demonstrates very vividly the working principles of a heat engine. These principles without the mathematics involved can be simply explained to 'older' (grade 4 and up) children. The use of a popular toy to illustrate scientific principles may incite many of these children to become permanently attracted to the excitement of science and to pursue careers in science and engineering when they grow up. For high school and college students, deriving the principles involved in the action of the toy will be a good exercise in understanding the practicalities of thermodynamics and mechanics. The working of this engine has been analysed from these points of view. The processes described below are suitable as laboratory experiments in physics, engineering and chemistry to illustrate these topics as well as to show how pertinent parameters can be obtained from experimentally acquired data by curve fitting methods.

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The working of the toy

The bird is made of glass. There are roughly three compartments: (i) an approximately spherically shaped bulb at the bottom, connected by (ii) a narrow length of glass tube stem to (iii) a similar spherical bulb at the top. The connecting tube extends to almost the bottom of the lower bulb as illustrated in figure 1. The lower bulb is partially filled with a red liquid (Freon 11). This quantity of liquid fills the length of the connecting glass tubing with enough of it remaining in the lower bulb to still immerse the glass tube. A small protrusion, shaped like a beak, is made of fibrous material and is attached to the upper bulb. The glass assembly is hinged on a support frame such that when the bird is in an upright position the centre of gravity of the bird-and-liquid is below the hinging points. Students familiar with mechanics will recognize that this is the mechanically stable equilibrium position.

Before the bird dips to 'drink', the liquid must first rise up the tube. The reason for the liquid to rise will be described below. As the liquid continues to rise, so does the centre of gravity. A point is finally reached such that the centre of gravity of the assembly is above the hinges. The system is now at an unstable equilibrium position and the top-heavy bird tips over to 'drink' from a glass of water placed before it. This tipping over causes the liquid column in the connecting glass tube to drain back into the lower bulb, bringing the centre of gravity back to below the hinging points. The bird is now back to its stable equilibrium position and the cycle repeats.

In order to understand the bird's drinking action, it is necessary to know that residual air is removed from the bird figure by pre-pumping before Freon 11 is introduced and the bird sealed. The vapour pressure of Freon 11 is sensitive to temperature. The drinking process begins at an initial ambient temperature T_1 . The compartments not filled with liquid are saturated with the vapour of the liquid at T_1 . If the temperature of the upper

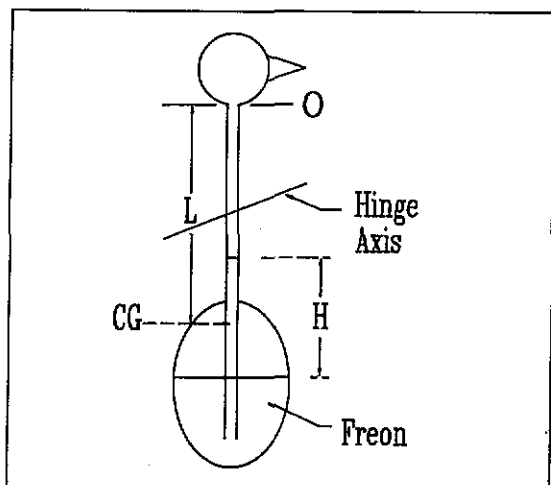


Figure 1. Schematic diagram of the drinking bird toy.

bulb is decreased to T_2 , the vapour pressure in it decreases. The higher pressure in the bottom bulb will drive a column of liquid Freon up the tube, compressing the vapour in the top bulb. More vapour will condense into liquid, so that the volume above the liquid in the upper bulb remains saturated with vapour at T_2 . Because of the rise in the level of liquid Freon, the vapour pressure in the lower bulb drops below the saturated vapour pressure at T_1 . Heat energy now flows in from outside the lower bulb (the surroundings being an infinite heat reservoir) to evaporate more Freon to maintain a saturated vapour required by this temperature.

The engine

The bird does not move initially. To start the drinking cycle, the beak of the bird is immersed in a glass of water, mimicking the drinking action. The act of 'drinking' raises the centre of gravity of the bird, i.e. work is being done against gravity. If the energy required to do this work were not supplied externally, this would be a perpetually moving engine, in violation of the second law of thermodynamics.

If one were to examine the upper portion of the bird where the beak is attached, one would discover that it is covered by a thin layer of webby, spongy material connected to the fibrous beak which soaks up water by capillary action. As a result, the top of the bird behaves like a sponge or the wet bulb in a dry-and-wet thermometer. Dependent on ambient relative humidity, the temperature of the upper part of the bird would have the temperature of a wet bulb thermometer

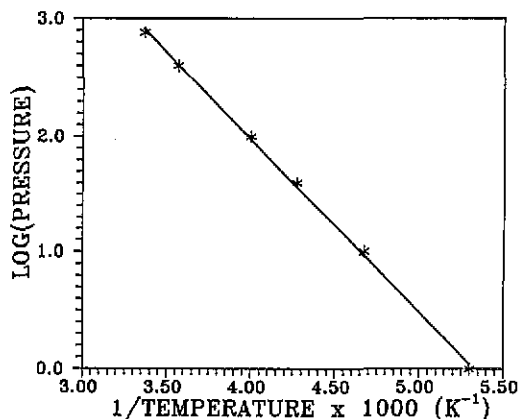
under the same ambient conditions. This temperature T_2 is lower than ambient temperature T_1 due to the continuous evaporation of water from its surface. This temperature drop causes a drop in vapour pressure in the upper half of the bird, vapour pressure in the bottom remaining high because of the higher temperature.

At ambient temperature of 22°C and relative humidity of 50%, the temperature difference between the dry and wet bulbs is 6.5 K [1]. The variation of vapour pressure with temperature is given by the Clausius-Clapeyron equation [2]. Alternatively, available data can be fitted into the following empirical equation [1]:

$$\log_{10} p = -\frac{a}{T} + b \quad (1)$$

where p is vapour pressure in mm of mercury, T is temperature in K, and a and b are constants. Freon 11 (CCl_2F , trichlorofluoromethane) has vapour pressures of 1, 10, 40, 100, 400 and 760 mm at temperatures of 188.7, 214.0, 234.0, 250.0, 279.8 and 296.7 K respectively [1]. These data are plotted in figure 2. From this plot, the constants a and b in equation (1) are determined to be $a = 28\,531 \pm 431 \text{ K}^{-1}$ and $b = 7.9 \pm 0.1$. At 22°C , a temperature change of 6.5 K results in a change in vapour pressure of 175 mm of mercury. The density of Freon 11 at 25°C is $1.467 \times 10^{-3} \text{ kg m}^{-3}$. 175 mm of mercury is equivalent to 162.2 cm of Freon 11. This change in pressure is enough to cause the Freon column in the bird to rise to drive it into motion. In the language of thermodynamics, the bird is a heat engine. It is absorbing heat energy from a heat reservoir at a higher temperature (bottom bulb) and discharging it to a heat reservoir at a lower temperature (upper bulb). In this process, mechanical work is extracted.

Figure 2. The logarithm of Freon 11 vapour pressure versus $1/T$.

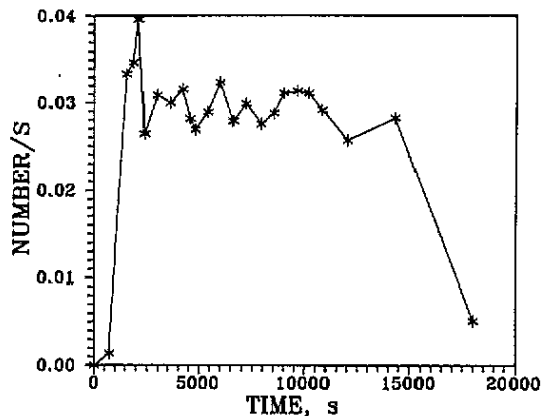


The experiment and results

The bird was originally stationary and a known quantity (2 ml) of water was slowly introduced into the fibrous material in the beak of the bird. The evaporation of the water induced a drop in temperature, which in turn initiated the series of events described above, with the bird dipping down and then rising again. The 2 ml of distilled water was applied a few drops at a time using a medicine dropper that has a 1 ml graduation. Each ml contains 20 drops of water. When the bird engine was first given a few drops, the Freon 11 column slowly rose. It took over ten minutes for the first cycle to complete. More water drops were slowly added. As the wet area of the webby, spongy material covering the top spherical bottle increased, the speed at which the bird dipped increased. Water drops were at first applied to other parts of the webby, spongy surface in addition to the beak. As the experiment progressed, only the beak received water drops. This accounted for the higher dipping rate at the beginning. For most of the experiment, the rate was about two dips per minute. It took about three hours to apply the 2 ml of water. It could not be applied faster because as the beak became saturated, any additional drops would just drip away and were wasted. The engine finally stopped when all the water had evaporated. $N(t)$, the bird's 'drinking' rate, was calculated by recording the time it took the bird to execute two complete dipping cycles. Such recordings were done at intervals of roughly ten minutes. These results are shown in figure 3.

To evaluate the mechanical work that is done by the toy when the column of liquid is raised to the top before being allowed to drain back, the locations of the centre of gravity of the toy at the

Figure 3. The rate of dipping (number per second) versus time, t .



beginning of the cycle when the column begins to rise and when the column is at its maximum height have to be determined. The following experiments were performed to obtain these positions.

After the first dip, it was noticed that the liquid in the column did not drain completely back to the bottom bottle. Instead, the second cycle began with a column about 3.5 cm high. The position of the centre of gravity was denoted by CG at the beginning of the second cycle, at a distance L measured downwards from a reference point O, chosen to be the top of the glass tube where it joins the upper bulb. H was the height of the liquid column, measured from the liquid level of the bottom bottle (see figure 1). Similarly, the position of the centre of gravity CG' was at a distance L' when the height of the liquid column was H' , about 11.5 cm just before the bird began to tip and the liquid drained back down. The mechanical work done by this heat engine is equal to the difference in potential energy of the two configurations.

Because of the vertical axial symmetry of the toy, the locations of CG and CG' moved along this axis of symmetry when the height of the liquid column changed. When this height was H , the toy's moment of inertia about an axis passing through CG and parallel to its 'drinking' axis of rotation is $I_{H,0}$. Distances were measured from the same reference point described above where L and L' were defined. A new axis of rotation parallel to the normal 'drinking' axis can be introduced at a distance x from the point O. The new moment of inertia, $I_H(x)$ about the new axis is given by [3]

$$I_H(x) = I_{H,0} + M(L-x)^2 \quad (2)$$

where M is the mass of the toy bird. If the bird is displaced an angle θ from the vertical and released, it will begin to oscillate about this axis as a compound pendulum under the restoring force arising from the action of gravity on the bird's centre of gravity. For a small angle of oscillation θ , this force results in a moment that is shown to be $Mg(L-x) \sin \theta = Mg(L-x)\theta$, where g is the constant of gravitational acceleration. The equation of motion is

$$I_{H,0} \ddot{\theta} = -Mg(L-x)\theta. \quad (3)$$

The period of this oscillation is given by

$$\begin{aligned} \tau &= 2\pi \sqrt{\frac{I_H(x)}{Mg(L-x)}} \\ &= 2\pi \sqrt{\frac{(I_{H,0}/M) + (L-x)^2}{g(L-x)}} \end{aligned} \quad (4)$$

Because $I_{H,0} = Mk_H^2$, where k_H is the radius of gyration for an axis through the centre of gravity of the bird, the period of oscillation τ simplifies to

$$\tau = 2\pi \sqrt{\frac{k_H^2 + (L-x)^2}{g(L-x)}} \quad (5)$$

The period of oscillation at each position x was determined by recording the time it took the bird to complete a certain number of swings. The time was measured using a digital wrist-watch that had a stop-watch mode. The x positions were measured on a strip of ruled paper taped on the glass stem (the bird's neck). The results are shown in figure 4.

It can be seen from equation (5) that, as x increases, the period decreases and then increases again; a minimum occurs at $x = x_{\min}$, and thus $L = k_H + x_{\min}$. If k_H is known, L can be determined.

By repeating the same experiment when the liquid column is at its maximum height H' , the location of the centre of gravity in this configuration, L' , can also be determined in terms of x_{\min} and $k_{H',0}$. The work done to raise the column of liquid is given by $W = Mg(L - L')$. However, the values of both these radii of gyration are not known, and the location where the minimum oscillation period occurred was not well defined, as can be seen from figure 4. L or L' could not be determined using this approach.

Equation (5) cannot be further linearized to facilitate data analysis to determine the period of this compound pendulum. A nonlinear least-squares curve fitting computer program† was then used to fit the data of figure 4 to equation (5)

to determine the quantities (k_H^2, L) and $(k_{H'}^2, L')$ of the two liquid column configurations from the experimental data. In nonlinear least-squares curve fitting, initial values of k_H and L are used in equation (5) to calculate τ at each value of x . The differences of the calculated and the experimentally measured values are called the residuals. The sum of the squares of the residuals is evaluated. The values of k_H and L are adjusted by the computer program and a new sum of squares of the residuals is calculated. The values of k_H and L that give the minimum residual squares are the answers sought.

For the initial height of the column after the first 'drink', the radius of gyration k_H^2 was $23.9 \pm 2.5 \text{ cm}^2$ and the centre of gravity was at a distance $L = 6.48 \pm 0.04 \text{ cm}$ from the reference point O. For the final height of the liquid column before tipping occurred, these quantities were $29.9 \pm 1.7 \text{ cm}^2$ and $5.43 \pm 0.02 \text{ cm}$ respectively. The calculated oscillation periods were also plotted in figure 4. The agreement is excellent. Therefore $L - L' = 1.05 \pm 0.01 \text{ cm}$.

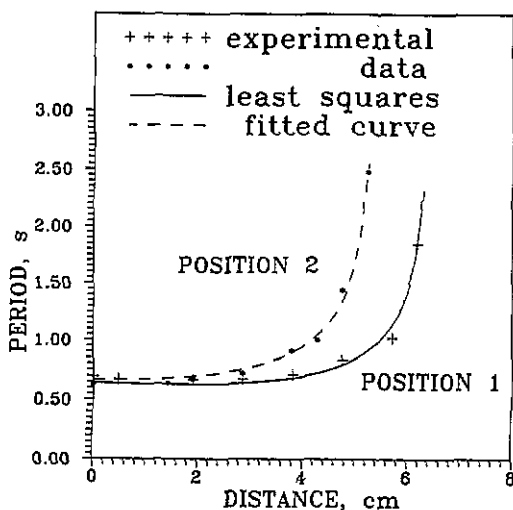
Discussion

The dry and wet bulb temperatures were 21.11°C and 17.8°C on the day the experiment was performed. The change in the centre of gravity due to the change in the liquid column was $1.05 \pm 0.01 \text{ cm}$, and the mass of the bird was measured accurately using an electronic balance to be $22.7065 \pm 0.0001 \text{ g}$. The work done in one cycle lifting the liquid was $W = 2.34 \pm 0.02 \text{ mJ}$.

The number of work cycles executed by the bird engine when 2 ml of water was evaporated was obtained by integrating the $N(t)$ rate curve in figure 3. This was done using the simple trapezoidal rule. The value obtained was 450. A more accurate number could be obtained by wiring an electronic counting circuit using simple components to count the total number of oscillations performed by the bird during the whole experiment. Nevertheless, the current method demonstrated to the students a way to obtain the total quantity from the rate of change of a quantity. The total useful mechanical work was $2.34 \times 10^{-3} \text{ J} \times 450 = 1.05 \text{ J}$.

The maximum theoretical efficiency of a heat engine operating between two heat reservoirs is

Figure 4. The period of oscillation at different values of x .



† Nonlinear least-squares curve fitting computer programs are available commercially for mainframe and desk-top computers and even from public domain shareware sources. The least-squares curve fitting described in this paper can be done manually by trial and error using a commercial spreadsheet such as Lotus-123.

given by $\Delta T/T = 3.33/294.1 = 0.011$. Since at least 4.572×10^3 J has flowed out of the hot reservoir to vaporize 2 ml of water, the ratio of the total useful mechanical work (1.05 J) to this quantity of heat energy is the efficiency of the working heat engine. The efficiency is 0.0002, considerably less than the ideal efficiency of 0.011 from thermodynamic considerations. The drinking bird heat engine is working at only 2% efficiency.

In each cycle, just before the liquid drains back to the lower bottle, most of the vapour is in the bottom bottle, where heat energy has flowed in from the surroundings. When the Freon drains back down, most of this vapour is transferred directly to the upper bottle, i.e. flows to the cold heat reservoir which is at the temperature of the wet bulb thermometer. As a result, most of the heat energy absorbed from the hot heat reservoir flows directly to the cold heat reservoir without doing any mechanical work. In fact, from our result, 98% of the heat energy is wasted. This is so because of the bird engine's design. A more 'clever' design, e.g. one in which a higher liquid column is produced before the Freon drains back, would no doubt improve the working efficiency of this heat engine, but thermodynamics says that even then it cannot exceed the theoretical efficiency of 0.011.

Conclusion

The drinking bird is not a perpetual motion machine. Experiments were done using it to demonstrate the thermodynamic principles of a heat engine. These experiments also demonstrated important techniques employed in scientific research, e.g. obtaining information from published sources, using linearization to analyse a set of data to extract the required parameters, obtaining data using simple yet practical procedures such as a stop-watch to measure the period of vibration of a mechanical system, to measure the rate of happening of an event, and to use nonlinear least-squares curve fitting techniques to extract information from a nonlinearizable data set, and graphical integration to obtain a quantity from the rate of change of this quantity.

References

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- [2] Atkins P W 1986 *Physical Chemistry* 3rd edn (New York: Freeman) p 141
- [3] Feynman R P 1963 *Feynman Lectures on Physics* vol 1 (New York: Addison-Wesley) ch 19

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